Assessment of Uncertainties of Measurement

Supplementary Example 2

Calculation of the uncertainty of a balance calibration

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The measurement

A 50 g capacity electronic balance, with a digital scale reading to a least count of 0.1 mg, has been examined for repeatability and departures from nominal scale values using methods based on *The Calibration of Weights and Balances*, Edwin C Morris and Kitty M K Fen

Note: Any internal zero-tracking systems must be disabled prior to carrying out a balance calibration.

Repeatability

The repeatability examination consists of ten repeated weighings of a mass without making any interim adjustments to the initial zero (no-load) setting of the balance, including automated zero-tracking. The individual differences between the ten "no-load" and the ten "laden" balance readings are calculated and the standard deviation of these differences is calculated. This examination is carried out at approximately half capacity and full capacity. The minimum permissible assigned value for standard deviation is equal to one third of the scale resolution. For this balance, the values for standard deviation (σ), were both calculated as 0.042 mg.

Departures from nominal scale value

The departures from nominal scale values are determined using a double weighing technique and calibrated OIML Class F_1 weights. The balance is prepared by running its automated internal calibration sequence (where provided) and disabling any zero-tracking system. The measurement sequence involves; zero (no-load) reading, place standard mass on to balance, take reading with standard mass on balance, remove and replace mass on to balance, repeat reading with standard mass on balance, remove standard mass from balance and repeat zero (no-load) reading. This procedure is carried out for a series of weighings made at intervals of one tenth balance capacity, and corrections to balance readings are calculated.

The model

The basic measurement model is -

$$C_i = M_i - (r_i - z_i)$$

Where, for the *i*th scale value:

- C_i is the calculated correction
- M_i is the calibrated value of the standard mass
- r_i is the mean of two repeated readings with mass on balance
- z_i is the mean of two no-load (zero) readings

The input data

For calibration at a scale value of 45 g, a combination of three standard masses are required. For this calibration, the relevant data are:

Mass	Value
1	20.000088 g
2	19.999995 g
3	5.000030 g
Total (M_i):	45.000113 g

Observations

1st zero reading	0.0000 g
1st reading of standard mass	45.0003 g
2nd reading of standard mass	45.0003 g
2nd zero reading	0.0001 g

Calculations

Mean zero reading (z_i)
= 0.00005 g

Mean reading on standard mass (r_i)
= 45.00030 g

Correction,

$C_i = M_i - (r_i - z_i)$
$= 45.000113 - (45.00030 - 0.00005)$ g
$= -0.000137$ g
$= -0.1$ mg
(rounded to least count of balance scale)

Uncertainty components and their evaluation

There are four major uncertainty sources to be considered in the evaluation of scale corrections (note that all sensitivity coefficients (c_i) are unity):

- Uncertainty of the calibrated value of the standard mass
- Uncertainty due to instability of the standard mass

- Uncertainty due to least count of balance scale
- Uncertainty due to the balance repeatability error.

Uncertainty of the standard mass value

There is an assumed correlation between systematic errors in the calibration values for individual masses used in combination. Therefore, the standard uncertainty for the standard mass is calculated by arithmetic addition of the individual standard uncertainties for each component mass of a combination. This combined uncertainty has a normal distribution. For this calibration, the relevant data for expanded uncertainty and 95% coverage factor (*k* value) are taken from the calibration report for the masses. The calculation of standard uncertainty and degrees of freedom for the combined masses is:

Mass	Value (g)	U_{95} (mg)	*k* value	*u* (mg)	ν
1	20.000088	0.019	2.0	0.0095	50
2	19.999995	0.019	2.0	0.0095	50
3	5.000030	0.009	2.1	0.0043	20
Totals:				0.0233	120

$$u_{MASS} = 0.0233 \text{ mg}$$
$$\nu_1 = 120$$

Uncertainty due to instability of the standard mass

The standards are relatively new and there is no substantial calibration history on which to base an estimate of stability. Experience with similar weights from the same manufacturer is that values remain stable within 10% of maximum permissible error over the period between calibrations.

For these OIML Class F1 weights, the maximum permissible errors are 0.25 mg for each 20 g mass and 0.16 mg for the 5 g mass. Instability is then estimated to be no greater than 0.025 mg for each 20 g mass and 0.016 mg for the 5 g mass. Instability is assigned a rectangular distribution and the standard uncertainties are:

$$u_{INSTABILITY\ (20g)} = u_3 = 0.025/\sqrt{3}$$
$$= 0.0144 \text{ mg} \quad \text{(20 g weights)}$$

$$u_{INSTABILITY\ (5g)} = 0.016/\sqrt{3}$$
$$= 0.0092 \text{ mg} \quad \text{(5 g weight)}$$

Because the estimate of the semi-range values are based on generic information rather than on specific data, confidence in the values is estimated to be no greater than about 75% (the relative uncertainty is 25%). There is a 1 in 4 chance that the nominated semi-range of the uncertainty for each mass is an underestimate. The number of degrees of freedom for each mass are therefore calculated as:

$$\nu_2 = \nu_3 = \nu_4 = 4^2/2 = 8$$

Uncertainty due to scale resolution

For this calibration, the least count of the balance scale is 0.1 mg. Assuming that the scale reading is rounded (not truncated) to the nearest 0.1 mg, then there is a rounding error of up to 0.05 mg. This rounding error is assigned a rectangular distribution with a semi-range of 0.05 mg. The standard uncertainty is:

$$u_{RESOLUTION} = 0.05/\sqrt{3}$$
$$= 0.0289 \text{ mg}$$

Because the semi-range describes absolute limits, the number of degrees of freedom (ν_2) are infinite. For the purposes of calculation, a large number is substituted for infinity:

$$\nu_5 = 1.00\text{e}+10$$

Uncertainty due to repeatability of the balance

The repeatability of the balance has been examined, and its standard deviation (σ), calculated from ten repeated readings, is 0.042 mg. This is a pre-characterised, normally distributed, component of uncertainty relating to instrument performance. For the purpose of evaluating scale corrections, double weighings are carried out. In combination with the pre-characterised standard deviation, the number of weighings is used to derive the standard uncertainty in the same manner as used to calculate a conventional ESDM:

$$u_{REPEAT} = \sigma/\sqrt{2}$$
$$= 0.042/\sqrt{2}$$
$$= 0.0297 \text{ mg}$$

The number of degrees of freedom (ν_3) are based on the number of readings (n = 10) used to calculate the standard deviation of the instrument. The number of degrees of freedom are:

$$\nu_6 = n-1$$
$$= 9$$

Component	Units	Dist.	*U* or *a*	Divisor	ν_i	$u(x_i)$	c_i	$\|c_i\|u(x_i)$ $=u_i(y)$	$u_i^2(y)$	$u_i^4(y)/\nu_i$
u_{MASS}	mg	Normal	0.0233	1*	120	0.0233	1	0.0233	0.00054	2.456e-9
$u_{INSTABILITY}$ (20g)	mg	Rect.	0.0250	1.7321	8	0.01443	1	0.01443	0.00021	5.425e-9
$u_{INSTABILITY}$ (20^+g)	mg	Rect.	0.0250	1.7321	8	0.01443	1	0.01443	0.00021	5.425e-9
$u_{INSTABILITY}$ (5g)	mg	Rect.	0.0160	1.7321	8	0.00924	1	0.00924	0.00009	9.102e-10
$u_{RESOLUTION}$	mg	Rect.	0.0500	1.7321	1.00e+10	0.02887	1	0.02887	0.00083	6.944e-17
$u_{REPEATABILITY}$	mg	Normal	0.0420	1.4142	9	0.02970	1	0.02970	0.00088	8.644e-7
			Sums						0.00276	1.007e-7
			Combined Standard uncertainty, $u_c(y)$						0.05254 mg	
			Effective degrees of freedom, ν_{eff}						75.7	
			Coverage factor, k=Student's t for ν_{eff} and CL 95%						1.99	
			Expanded uncertainty, $U=k\,u_c(y)$						0.1046 mg	

* A divisor of 1 is used as this component has already been reduced to a standard uncertainty when the mass uncertainties were combined.

Table 1 Uncertainty calculations for scale correction at 45 g

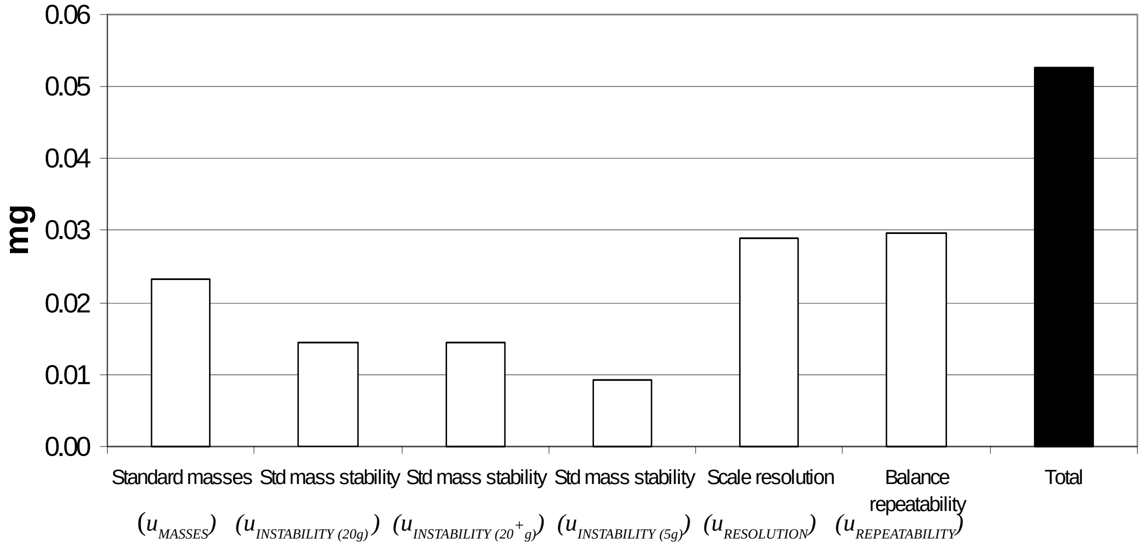

Figure 1 Comparison of magnitudes of standard uncertainty components

The reported results

Standard deviation at 20 g* = 0.042 mg Standard deviation at 50 g = 0.042 mg

Nominal reading (g)	Correction (mg)	U_{95} (mg)
5	0.0	0.09
10	0.0	0.09
15	0.1	0.09
20	0.0	0.09
25	0.0	0.09
30	- 0.1	0.10
35	0.0	0.10
40	- 0.1	0.10
45	- 0.1	0.10
50	- 0.2	0.10
	Max. absolute: 0.2mg	

*Repeatability examined at 20 g rather than at exactly half-load (25 g) to avoid using a combination of masses.

Limit of Performance of the balance

The Limit of Performance is the maximum expected error for a single weighing without applying corrections for departures from nominal scale value or built-in decade masses. The Limit of Performance is calculated as the sum of components for repeatability, maximum departure from nominal scale value and its associated uncertainty, and maximum correction for any built-in decade masses and its associated uncertainty.

For this calibration, the absolute values of correction plus associated uncertainties are:

Nominal reading (g)	**Absolute value of correction plus uncertainty (mg)**
5	0.09
10	0.09
15	0.19
20	0.09
25	0.09
30	0.20
35	0.10
40	0.20
45	0.20
50	0.30

The relevant data for evaluating Limit of Performance is:

Component	Value (mg)	Multiplier	Input value (mg)
Balance repeatability	0.042*	2.26	0.095
Max. scale corr'n + uncert.	0.30	1.0	0.30
Built-in masses + uncert.	0.0+	1.0	0.0
		Total:	0.395 mg

* The value of 0.042 mg for the balance repeatability is expressed as a standard deviation. Its multiplier of 2.26 is equal to the *k* value for n=9 at a CL of 95%.

+ In this balance there are no built-in decade masses.

Note: The correction and associated uncertainty for the balance's internal calibration mass is not included because any associated errors have been incorporated into the scale correction values by carrying out the internal calibration sequence before commencing the calibration.

The Limit of Performance of the balance is ±0.4 mg.